TORTOISE V. ACHILLES, 4 MIND 278 (1895)

Alert readers will recall that we left the U.K.'s Ministry of Defense in a pickle over pictures of good looking men. In the quarters of some of us, that's okay; when others put 'em up, that's an offense against military morals and grounds for the old heave-ho. There's different sorts of stuff in the world; however, pushing stuff into this bag or that box is a bit trickier than first meets the eye. We leave the warriors at MOD to their philosophy and turn to Achilles, the warrior of whom Homer had nothing good to say and Zeno didn't buff up his rep either. In our imagination Zeno has fleet Achilles, flailing away forever, halving the distance to the finish line, but never quite snagging the laurel that was his due.

Charles Dodgson, the eminent Oxford mathematician, also poked fun at Achilles; a link to his paradox is on line at www.lewiscarroll.org/logic.html. He published the article *What the Tortoise said to Achilles* (1895) in *Mind*, which is not in your local library, so the web is your best bet. Dodgson – and many turn-of-the-century brains – put their shoulders to sorting out exactly what apparatus had to be brought to bear when a problem of infinite steps or regression came into focus. At the same time men and women were looking for the most elegant but complete solutions to logical questions, in notation.

For every spare effort in system building, there was a system buster, insisting that there were more assumptions that had to be written down as beginnings; for every system buster, there was one more patch offered by the builders, to repair damage to their elegant notation. Dodgson asserted (in his fable cited above) that a well known rule of inference by the name of Modus Ponens (and rules of inference in general) must be made explicit; which was full employment for Achilles and a poke at system builders and their (increasingly voluminous) notations.

So how do lawyers and judges (the artisans of the law) rate, when their efforts at sorting out A *from* not-A are matched up against these trans-Atlantic thinkers? Don't we stumble, when we endlessly patch the apparatus, with no end in sight? Aren't our follies worth a fable or two? Take Erv the narcotics informant; he lies when he's with drug dealers and he claims to be telling the truth (juryside) about what he said and did when he was in the market for drugs.

The prosecutor urges that there are two venues at work in Erv's telling: in one venue the witness is privileged to lie; indeed it would be very unwise for Erv to tell dealers he works for the cops. In the other venue, he is sworn to tell the truth and subjected to cross examination so that third parties – judge and jury – may discern the truth of what he said to the dealers (and they to him) and (in court and in the presence of judge and jurors) what Erv says on the stand. We may say that there are two of them: Erv the lier on the street and Irv the proponent of truth in court. And to make things easier (or sloppier), a neutral decisionmaker sorts out what Irv was lying about when he said he was telling the truth about Erv's lies. Which is what the trial venue is all about.

Artisans do not call on the prosecutor to assume or ask for acceptance of his rule of inference. Effectively, this leaves the jury is free to discount Irv's proferred testimony, on its view of Erv's admitted lies. All of this puts Irv is at risk juryside, just as he was in the markplace. Bertrand Russell, however, constructed rules of inference (a whole theory's worth) which required events at the "Erv" (street) level to be segregated from events at the "Irv" (trial) level: through Russell's apparatus it was legal (logically of course) to ask if Irv was lying when he said he was being truthful today about the lies told yesterday. Posing that one was not paradox.

Russell offered his own apparatus – called the theory of types – because he said that (without his theory) the following mess was logically inevitable: one can start with a function L (which assigns the property L to input like "stuff Irv said"). Now take as input "Irv is lying" and stick that input into a notation like L(I). This is a way of putting L(I) back into the L function. In notation, posing L[L(I)] as a question reads: Is Irv lying when he says he's lying? He is if he is and he isn't if he is.

Pick your poison: the author of the *Principia Mathematica* says you need a robust apparatus to keep paradox out of reasoning and the author of *Alice in Wonderland* hints that, even with pages of rules of inference, we're at risk to reason forever without finality. And yet most juries do reach conclusions, which are tested in trial courts and on appeal. And then the Legislature and the Academy drive by and let loose their volleys.

Modern readers will recall our own Douglas Hofstadter's dense but charming *Gödel, Escher, Bach (1978)*, which contains literally endless

examples of regressive paradox. (I may be repeating myself.) From the artisan's point of view: we commit in advance to limited formal outcomes (true/false, guilty/not guilty, liable/defense verdict); we constrain ourselves to notation for our results. Since input and method determine result, the work is in getting the procedures right. This should ring a familiar bell; as artisans, we're an experiment-happy bunch, taking a millennium and a half of our Corpus Juris at one bite.

Outside of these interlocking venues – operating in court systems, and (in Erv's case) in systems of drug distribution – solving the Liar's Paradox has not been nearly as much fun. Paul the Apostle wrote this to Titus: "A prophet of their own says that Cretins are always liars." (1:12). This prophet was Epimenides and a lot of ink was spilled (at the turn of the last century) sorting out this one: if all statements that were untrue could be shoved into a box and that box contained only statements that were untrue, what about a statement that *this* statement was untrue? Was the last thing I said in the box?

Which is where the Tortoise and the artisans of the law may be in accord. Screening input requires some dry runs; and a finite number of dry runs of functions that tend to infinite regression – especially when experimenting with trial as venue – makes perfect sense. Why should what we do be so right? That's another question and a darn good thing to keep in *Mind*.

Apparatus

Author:	Peter J. Aschenbrenner
First published:	The Alaska Bar Rag
Revisions:	Minor.

Our Server Version:	5 - Tortoise v. Achilles, 4 Mind 278 (1895)